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TURNED IVORY POLYHEDRA

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ABSTRACT: In this paper a survey of nested ivory spheres and polyhedra made by turning in Europe between the 16th and 18th centuries is presented, different polyhedra are identified, and examples of related objects of known ivory collections are shown. These examples demonstrate the high level of geometrical knowledge of the old master turners who produced these objects.

Keywords: Geometry, Polyhedra, Nested spherical shells, Chinese ball, Turning, Turnery art, Ivory carving.

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1. INTRODUCTION

Nowadays souvenir shops in China sell "devil's work balls" or "puzzle balls", that is, multiple spherical nested shells made of jade, sandalwood or artificial ivory. During the Qing Dynasty, such multi-layer balls turned on a lathe were made of ivory with piercing and rich figurative carving (Figure 1). The manufacturing technology required radially drilled conical holes whose number is 12 or 14; 12 holes at the face centres of a regular dodecahedron, while 14 holes at the face centres of a cuboctahedron. For these Chinese balls, mainly the aesthetic appearance was important, and geometry was just something necessary to it. It is little known how their making started. The only thing that we know is that Chinese balls already existed in the 14th

Figure 1: A 12-layer Chinese ball with 14 holes, carved from a solid block of ivory, Qing dynasty, 19th century, inv. no. AO 1906-10, British Museum, London.

Figure 2: An ornamental 7-layer ivory ball with 12 holes, carved from a solid block of ivory, late 16th century, inv. Bg no. 202, Museo degli Argenti, Florence.

century. The book [1] made mention of a three-layer ball.

Similar ivory art objects appeared in Europe in the second half of the 16th century. Their manufacturing method was developed in Germany. The art of turning ivory was prosperous until the 18th century when with the appearance of porcelain the art of ivory carving started to decay. Contrary to the Chinese balls, the European balls have a smooth surface and a polyhedron-like shape and their beauty came from the perfect geometry (Figure 2). The polyhedral form shows a high level of geometrical knowledge of the artists, and this is also why the number of applied holes is not only 12 or 14, but many more (e.g. 6, 18, 20, 24, 26, 32). Another feature is that often radial spikes come through the holes from the core of the ball, but such spikes are missing from the Chinese balls.

In this paper, we present a survey of nested ivory spheres and polyhedra made by turning, and show examples of related objects of art of known ivory collections in Dresden, Vienna, Copenhagen, Florence, Oxford, Paris. We want to determine what kinds of polyhedra are present at these objects.

(a) (b)

Figure 3: Turned ivory goblets, Danish Royal Collections, Rosenborg Castle, Copenhagen: (a) inv. no. 2701, probably made in Nuremberg, 1600-1650, (b) inv. no. 2703, probably made in Nuremberg, c. 1650.

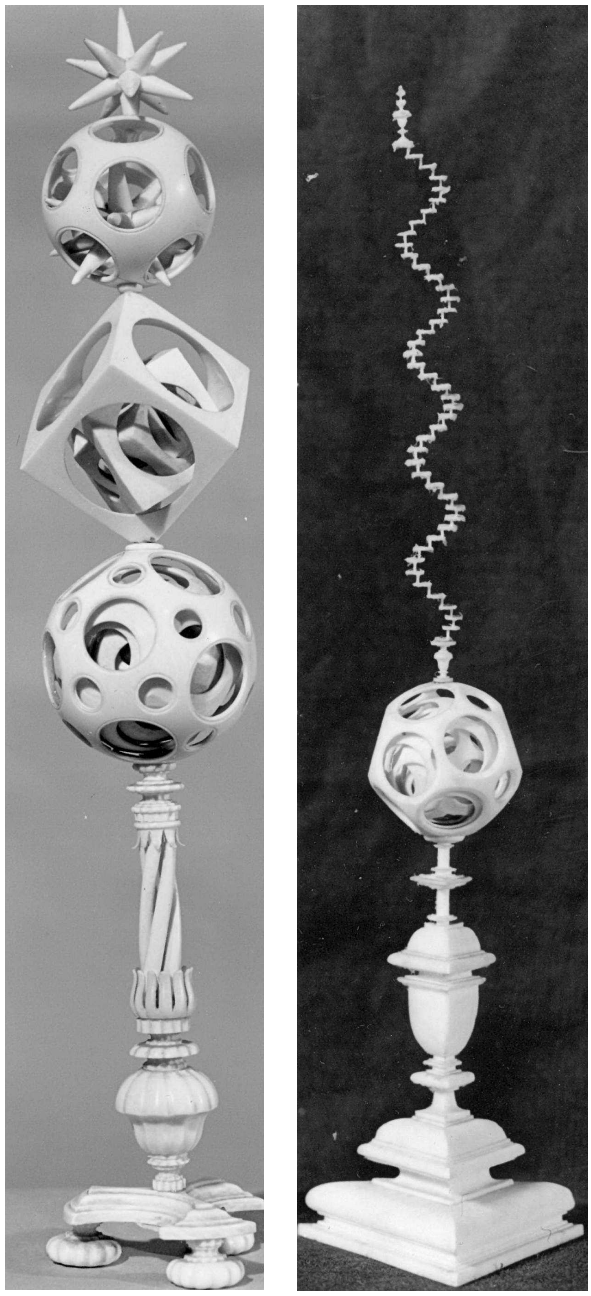
2. HISTORY

Ivory carvings were among the rarities collected by princes and wealthy people in Europe during the sixteenth and seventeenth centuries. Many of their places had a *Kunstkammer* or a *Wunderkammer* (cabinet for art or curiosities), where their treasures were displayed. The intention was to suggest the wealth and knowledge of the collector and to impress guests. That was also the age of exploration, when exotic materials and plenty of ivory were brought home from the discovered lands. The technique of forming ivory objects on a lathe by turning reached a high degree of complexity. French, Italian and German workshops produced thin, fascinating shapes from single blocks of ivory. Noblemen pursued turning as a hobby. Among the sovereigns who collected masterpieces of turning for their *Kunstkammer* and practiced the art themselves were Princes, Dukes, Grand Dukes, Electors, Kings, Holy Roman Emperors. They established turning chambers in their courts and invited the best turners to teach them and to produce objects for their collections. Turning was also a good tool to teach geometry. Some art historians think that the technique producing hollow spheres (contrefait spheres) including nested, multi-layer spheres and polyhedra by turning was invented by Giovanni Ambrogio Maggiore between 1574 and 1577. It is a fact, however, that he introduced contrefait spheres to the court in Munich in 1582, and Georg Wecker made a nested dodecahedron in Dresden in 1581. Multi-layer spheres and polyhedra usually occurred as parts of objects, for instance, decorations on the lid of a goblet as shown in Figure 3, but sometimes, they appeared as individual geometric objects as shown in Figure 4.

The ducal and royal *Kunstkammer* pieces formed invaluable collections by the 18th century, which contained also a great number of virtuoso turned ivory objects. The surviving pieces are kept together in different museums. The largest collections are the following: the Collection of the Saxon Electors in the Grünes Gewölbe, Dresden, Germany [2]; the Schloss Ambras Collection of the Habsburg Emperors in the Kunsthistorisches Museum, Vienna, Austria [3]; the Danish Royal Collections in the Rosenborg Castle, Copenhagen, Denmark [4]; the Medici Treasury in the Museo degli Argenti, Palazzo Pitti, Florence, Italy [5]. Additionally, there are several private and public museums where turned ivory master-pieces are exhibited. The Tradescant Collection is worth mentioning, which was not a ducal or imperial collection. John Tradescant and his son were court gardeners in England, and their collection became the foundation of the Ashmolean Museum, Oxford, UK, one of the first museums in Europe. We mention one place more: Conservatoire National des Arts et Métiers, Paris, France (CNAM Paris), where a cabinet is dedicated to the art objects of François Barreau (1731-1814), the best ivory and wood turner of the time, who, with virtuosity, produced intricate and fascinating turnings such as interlocked hollow spheres.

3. GEOMETRY

Turners of the Renaissance and of the baroque era knew well geometry in general and polyhedra in particular. Several books were



(a) (b)

Figure 4: Nested ivory spheres and polyhedra on a shaft, Museo degli Argenti, Florence: (a) inv. Bg no. 215, (b) inv. Bg no. 204, made in Nuremberg, probably in the workshop of the Zick family, 17th century.

published on regular polyhedra and their derivatives. This is why so easy to find examples of the five *Platonic solids* among ivory turnings: tetrahedron (Figure 5a), cube (Figure 5b), octahedron (Figure 5c), dodeca-

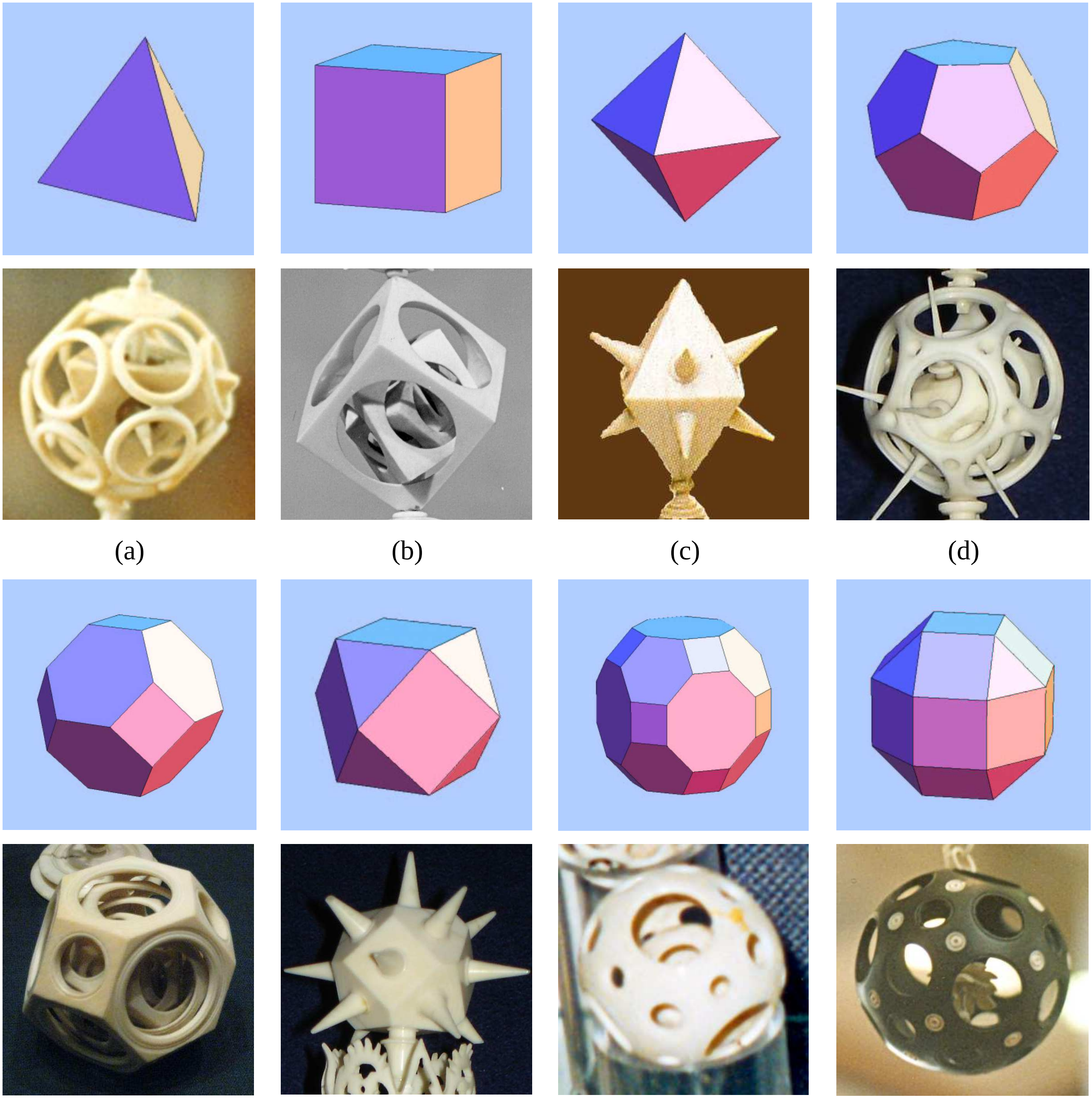
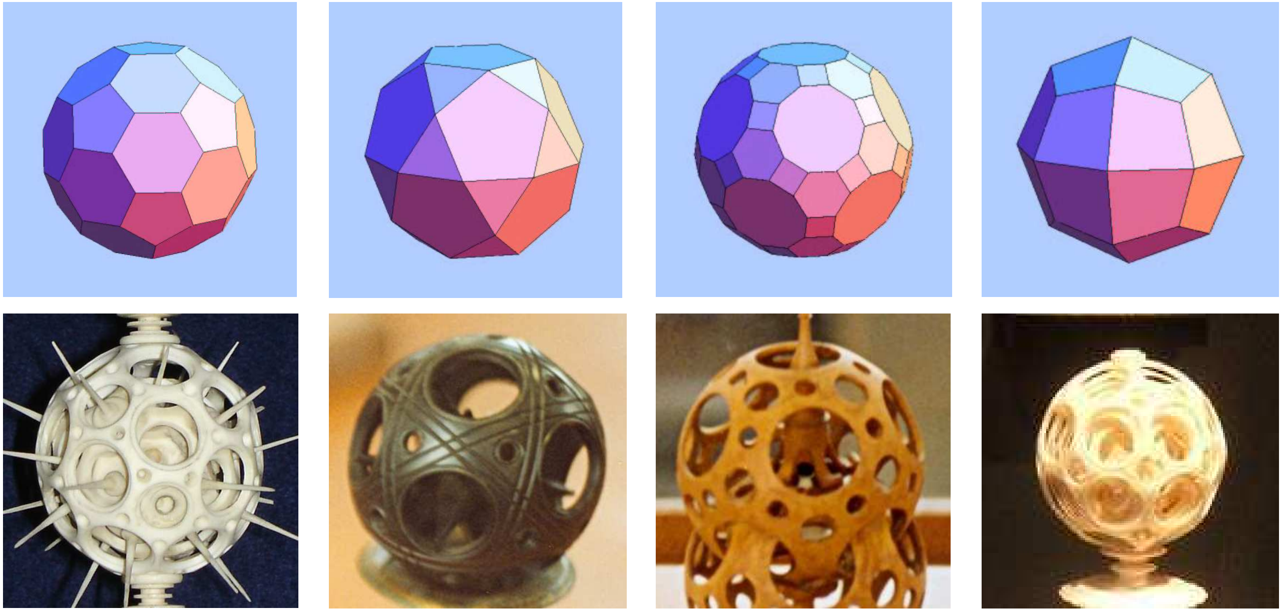


Figure 5: Some identified polyhedra. (a) Tetrahedron, François Barreau, around 1800, detail, inv. no. 104-23, Conservatoire National des Arts et Métiers Paris. (b) Cube, detail of the ivory object in Figure 4a. (c) Octahedron, detail of an ivory object of art, probably Dresden around 1600, inv. no. II 255, Grünes Gewölbe Dresden. (d) Dodecahedron, detail of the goblet in Figure 3a. (e) Truncated octahedron, turned ivory, German, around 1650, inv. no. KK 3617, Kunsthistorisches Museum Vienna. (f) Cuboctahedron, detail of the goblet in Figure 3b. (g) Great rhombicuboctahedron, probably German, first half of the 1600s, inv. no. Pl. CXX No. 242, Tradescant Collection, Ashmolean Museum, Oxford. (h) Rhombicuboctahedron, François Barreau, around 1800, ebony, inv. no. 104-2, Conservatoire National des Arts et Métiers Paris.



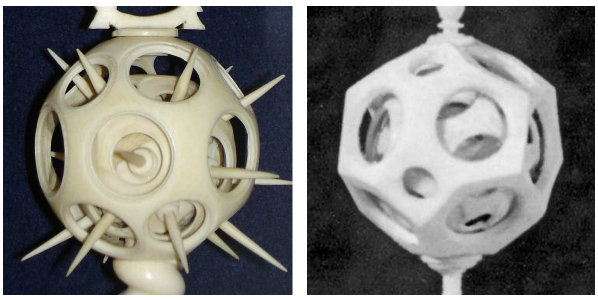
(i) (j) (k) (l)

Figure 5 (continued): (i) Truncated icosahedron, detail of the goblet in Figure 3a. (j) Icosidodecahedron, François Barreau, around 1800, ebony, inv. no. 104-57, CNAM Paris. (k) Great rhombicosidodecahedron, François Barreau, boxwood, around 1800, inv. no. 104-8, CNAM Paris. (l) Deltoidal icositetrahedron, Georg Friedel 1611-1619, detail, Grünes Gewölbe Dresden.

hedron (Figure 5d), icosahedron (not shown here).

The *Archimedean solids* are solid polyhedra whose all vertices are equal but not regular, and all faces are regular but not equal. In the late Renaissance, these solids were known and, for instance, Kepler gave a complete list of them in his *Harmonices Mundi* in 1619. There exist 13 Archimedean solids (and two additional infinite classes of prisms and antiprisms) from which we could find seven among the studied turnings: truncated octahedron (Figure 5e), cuboctahedron (Figure 5f), great rhombicuboctahedron (Figure 5g), rhombicuboctahedron (Figure 5h), truncated icosahedron (Figure 5i), icosidodecahedron (Figure 5j), great rhombicosidodecahedron (Figure 5k).

The *Catalan solids* are solid polyhedra whose all faces are equal but not regular, and all vertices are regular but not equal. Catalan solids are duals of the Archimedean ones. In the Renaissance, only some of them were known. We found two Catalan solids among ivory turnings: deltoidal icositetrahedron (Figure 5l), which is the dual of the Archimedean cuboctahedron, and the rhombic triacontahedron (not shown here). The fact that we could find only two is not surprising since the complete set of the 13 Catalan solids was shown by Eugène Catalan only in 1865.



(a) (b)

Figure 6: Ivory polyhedra different from regular and semiregular: (a) detail of a goblet, inv. no. 2748, Royal Danish Collection, Copenhagen, (b) detail of an object, inv. Bg no. 203, Museo degli Argenti, Florence.

The objects in Figure 6 do not belong to any of the above-discussed classes. The sphere in Figure 6a contains altogether 20 holes, and has D_{3d} symmetry. The polyhedron in Figure 6b (and also in Figure 4b) is probably derived from the reciprocal of the pentagonal Archimedean prism, that is, from a pentagonal Catalan bipyramid in a way, that the vertices of the pentagonal bipyramid are cut off with planes tangent to its insphere.

The arrangement of circular holes on the ivory spheres is in a close relationship with the problem of the minimum covering of a sphere with circles [6]. The reason for that is that the turner makes the internal layers with sideways cuts at the radial holes such that no material remains in the intersecting gaps. A cutting model for the sphere in Figure 5i can be, for instance, that in Figure 7.

Figure 7: Minimum covering of the sphere with 32 equal circles. A card model.

4. CONCLUSIONS

Studying ivory spheres and polyhedra made by master turners on a lathe in the 16th, 17th and 18th centuries, we discovered their geometrical background. From the studied material, in the actual turned ivory (wood) objects, we could identify five Platonic, seven Archimedean, two Catalan polyheda and two non-regular polyhedra as well.

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REFERENCES

[1] Cao Zhao, *Essential Criteria of Antiquities* (in Chinese), Nanjing, 1388. (*Chinese connoisseurship: the Ko Ku Yao Lun*, a translation made and edited by Sir Percival David, Faber, London, 1971.)

[2] Kappel, J. and Weinhold, U. *The New Grünes Gewölbe: Guide to the Permanent Exhibition.* Staatliche Kunstsammlungen Dresden, Deutscher Kunstverlag (2007).

[3] Haag, S. *Meisterwerke der Elfenbeinkunst* (Kurzführer durch das Kunsthistorische Museum, Band 8: Elfenbeinkunst, 191 S.), KHM, Wien (2007).

[4] Hein, J. and Kristiansen, P. *Rosenborg: A Guide to the Danish Royal Collections* (6th ed.), Rosendahls, Copenhagen (2005).

[5] Mosco, M. and Casazza, O. *The Museo degli Argenti: Collections and Collectors*, Giunti, Florence (2004).

[6] Tarnai, T. and Gáspár, Zs. Covering the sphere by equal circles, and the rigidity of its graph. *Mathematical Proceedings of the Cambridge Philosophical Society* **110**, 71-89 (1991).

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